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Title: Phenomenology of Light Systems Using R-Matrix Theory

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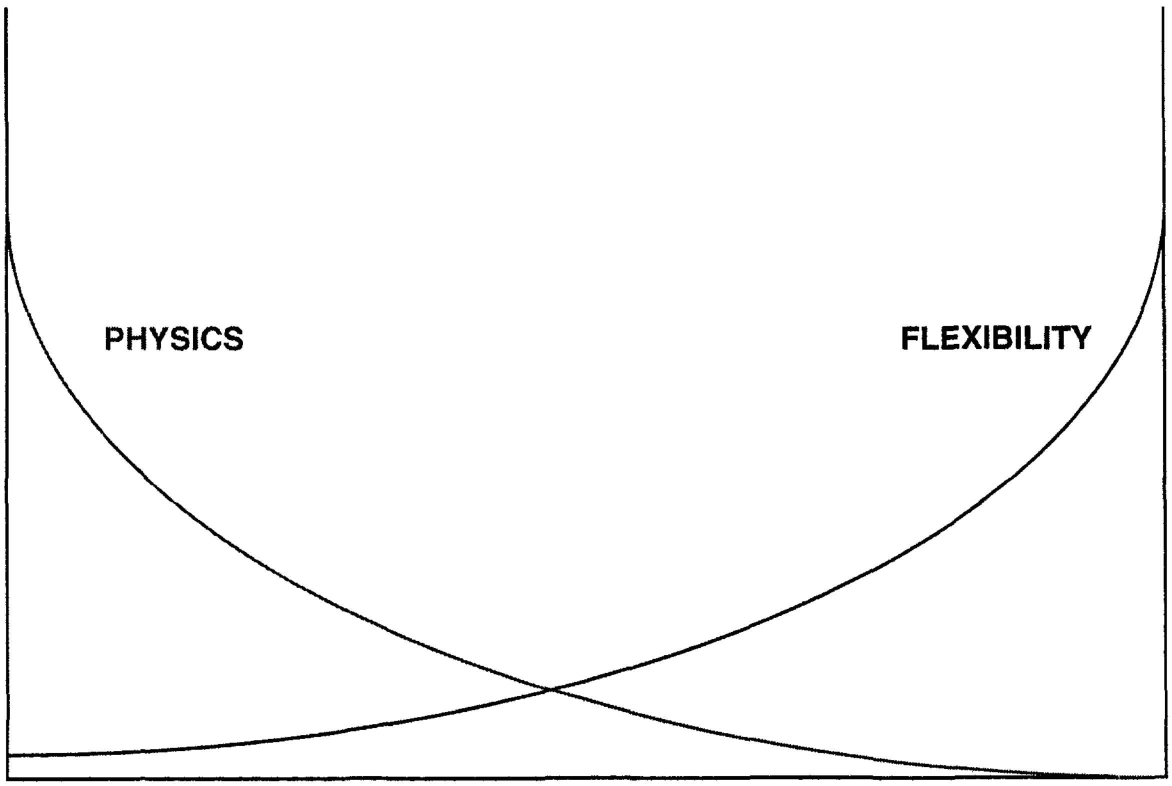
Los Alamos
NATIONAL LABORATORY

Talk to be given at the U. Erlangen, Germany on 4/9/01:

Phenomenology of Light Systems Using R-Matrix Theory

Over the last 30 years, a program of R-matrix analyses has been carried out at Los Alamos in the mass range $2 \leq A \leq 18$ that gives useful information about the light systems. These analyses provide data for astrophysics, as well as for neutronic and thermonuclear applications, and can yield important constraints from the experimental measurements on the nature of few-body interactions. Following a brief introduction to R-matrix theory, I will describe recent work on reactions in the A=2 (NN) and A=4 systems. Finally, some comments will be made about extending the theory to complex energies in order to obtain information about resonances and other types of S-matrix singularities.

METHODS FOR DATA EVALUATION



Microscopic Models:
phenomenology put into the N-N potentials; results are "data quality" for A=2,3

R-Matrix Theory:
some physical constraints, but flexible enough to fit most data for light systems

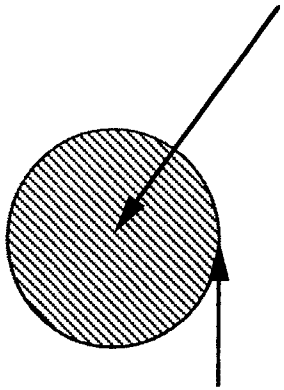
Generalized Least Squares:
almost completely mathematical; can fit any data

(Note: both talks will use different combinations of the same figures)

INTERIOR (Many-Body) REGION
(Microscopic Calculations)

$$\underbrace{H+\mathcal{L}_B}$$

compact, hermitian operator with real, discrete spectrum; eigenfunctions in Hilbert space



$$|\,\psi^+) = (H+\mathcal{L}_B - E)^{-1}\mathcal{L}_B|\,\psi^+)$$

SURFACE

$$\mathcal{L}_B = \sum_c |c)(c|\left(\frac{\partial}{\partial r_c} r_c - B_c\right),$$

$$(\mathbf{r}_c|c) = \frac{\hbar}{\sqrt{2\mu_c a_c}}\frac{\delta(r_c - a_c)}{r_c}[(\phi_{s_1}^{\mu_1}\otimes\phi_{s_2}^{\mu_2})_s^{\mu}\otimes Y_l^m(\hat{\mathbf{r}}_c)]_J^M$$

ASYMPTOTIC REGION
(S-matrix, phase shifts, etc.)

$$(r_{c'}|\psi_c^+\rangle = -I_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})S_{c'c}$$

Measurements

Green's Operator Approach to R-Matrix Theory

$$G^{NR} = (H + \mathcal{L}_B - E)^{-1} = \sum_{\lambda} \frac{|\lambda)(\lambda|}{E_\lambda - E}$$

(possible relativistic extension)

$$G^{\mathrm{Rel}} = [(p_1 + p_2)^2 + \mathcal{L}_B - s]^{-1} = \frac{2M\,|\lambda)(\lambda|}{s_\lambda - s}$$

$$R_{c'c} \equiv (c'|G^{\mathrm{Rel}}|c) = \sum_{\lambda} \frac{\gamma_{c'\lambda}\gamma^T_{\lambda c}}{(s_\lambda - s)/(2M)},$$

keeping in mind that

$$\frac{1}{2M}(s - M^2) \to E_{c.m.}$$

R $\rightarrow$ S Relations

$$R_B \rightarrow R_L = \left[1 - R_B(L - B)\right]^{-1} R_B,$$

$$L_c = a_c \frac{\partial O(r_c, k_c)}{\partial r_c} / O(r_c, k_c)\Big|_{r_c = a_c}.$$

The matching equation gives

$$T = O^{-1} R_L O^{-1} - F O^{-1}, \quad F = \mathrm{Im}(O),$$

with

$$S = 1 + 2iT$$

satisfying generalized unitarity, causality, etc., everywhere in the complex plane.

Physical Constraints in R-matrix Theory

Unitarity ($SS^\dagger = S^\dagger S = 1$):

Comes from R_B hermitian (real, symmetric); results in $\sigma_c^T = \sum_{c'} \sigma_{c'c}$, limits on $\sigma_{\max}^T$, and constraints linking S-matrix elements for different reactions.

Causality ($U^+(t) = 0$ for $t < 0$):

Comes from R_L having no poles on the physical sheet (other than bound states); results in limits on how fast phase shifts can change with energy.

Truncation of the Nuclear Partial Wave Series:

Comes from finite channel radii (short-ranged SI) and the Coulomb/angular-momentum barrier. Assures reasonable behavior of low-energy cross sections.

Built-In Symmetries of the Strong Interactions (SI):

- Conservation of J^π
- Time-reversal invariance (R, S symmetric)

Approximate Symmetries of SI:

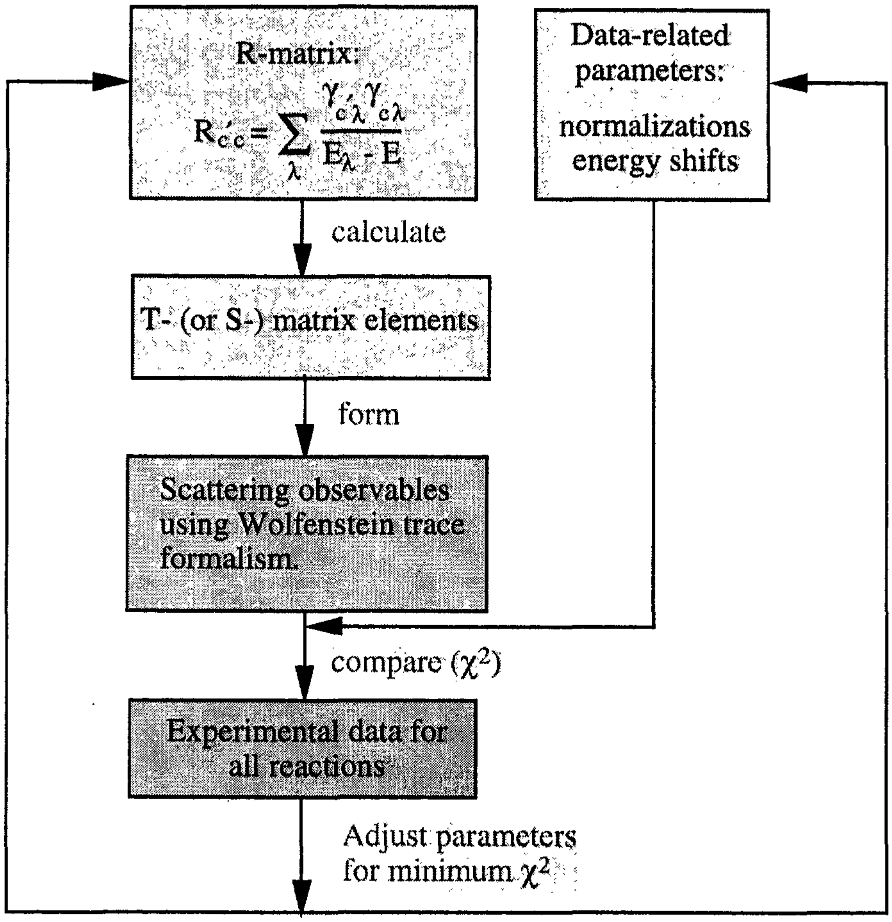
- Charge independence (charge symmetry)

Example: 2 X 2 S-Matrix in Stapp's Parametrization

$$S = \begin{pmatrix} \eta \exp(2i\delta_1) & i\sqrt{1-\eta^2}\exp[i(\delta_1+\delta_2)] \\ i\sqrt{1-\eta^2}\exp[i(\delta_1+\delta_2)] & \eta \exp(2i\delta_2) \end{pmatrix},$$

$$(0 \le \eta \le 1)$$

Energy Dependent Analysis Code



Capabilities and Features

1) Accomodates general (spins, masses, charges) two-body channels

2) Uses relativistic kinematics and R-matrix formulation

3) Calculates general scattering observables for $2 \rightarrow 2$ processes

4) Has rather general data-handling capabilities

5) Uses modified variable-metric search algorithm that gives parameter covariances at a solution.

Covariances

Input:

For measured observables $O_i = R_i N_n$, it is assumed that the R_i and N_n are mutually uncorrelated, so that

$$\begin{aligned}
\mathrm{cov}(R_i, R_j) &= (\Delta R_i)^2 \delta_{ij}, \\
\mathrm{cov}(N_n, N_m) &= (\Delta N_n)^2 \delta_{nm}, \\
\mathrm{cov}(R_i, N_n) &= 0,
\end{aligned}$$

and thus

$$\chi^2(\mathbf{p}) = \sum_{i,n} \frac{(R_i - O_i(\mathbf{p})/N_n)^2}{(\Delta R_i)^2} + \sum_n \frac{(N_n - 1)^2}{(\Delta N_n)^2}$$

Output:

First-order propagation of error gives at the solution

$$\mathrm{cov}\big(O_i(\mathbf{p_0}), O_j(\mathbf{p_0})\big) = \sum_{k.l} \left(\frac{\partial O_i}{\partial p_k} C_{kl} \frac{\partial O_l}{\partial p_l} \right)\Bigg|_{\mathbf{p}=\mathbf{p_0}}$$

with the parameter covariance matrix given by

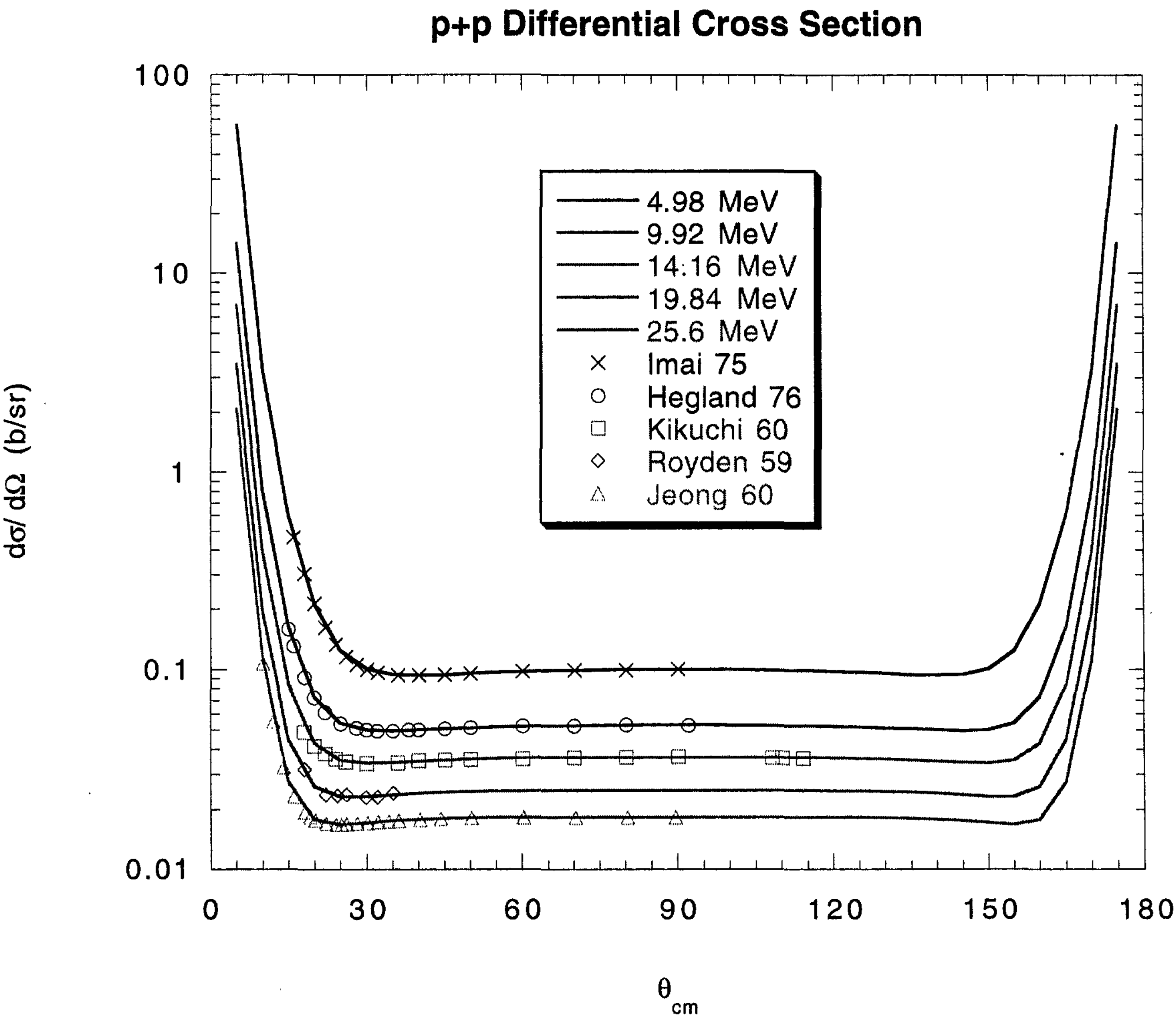
$$\begin{aligned}
C &= 2G^{-1}, \\
G_{kl} &= \frac{\partial^2 \chi^2}{\partial p_k \partial p_l}
\end{aligned}$$

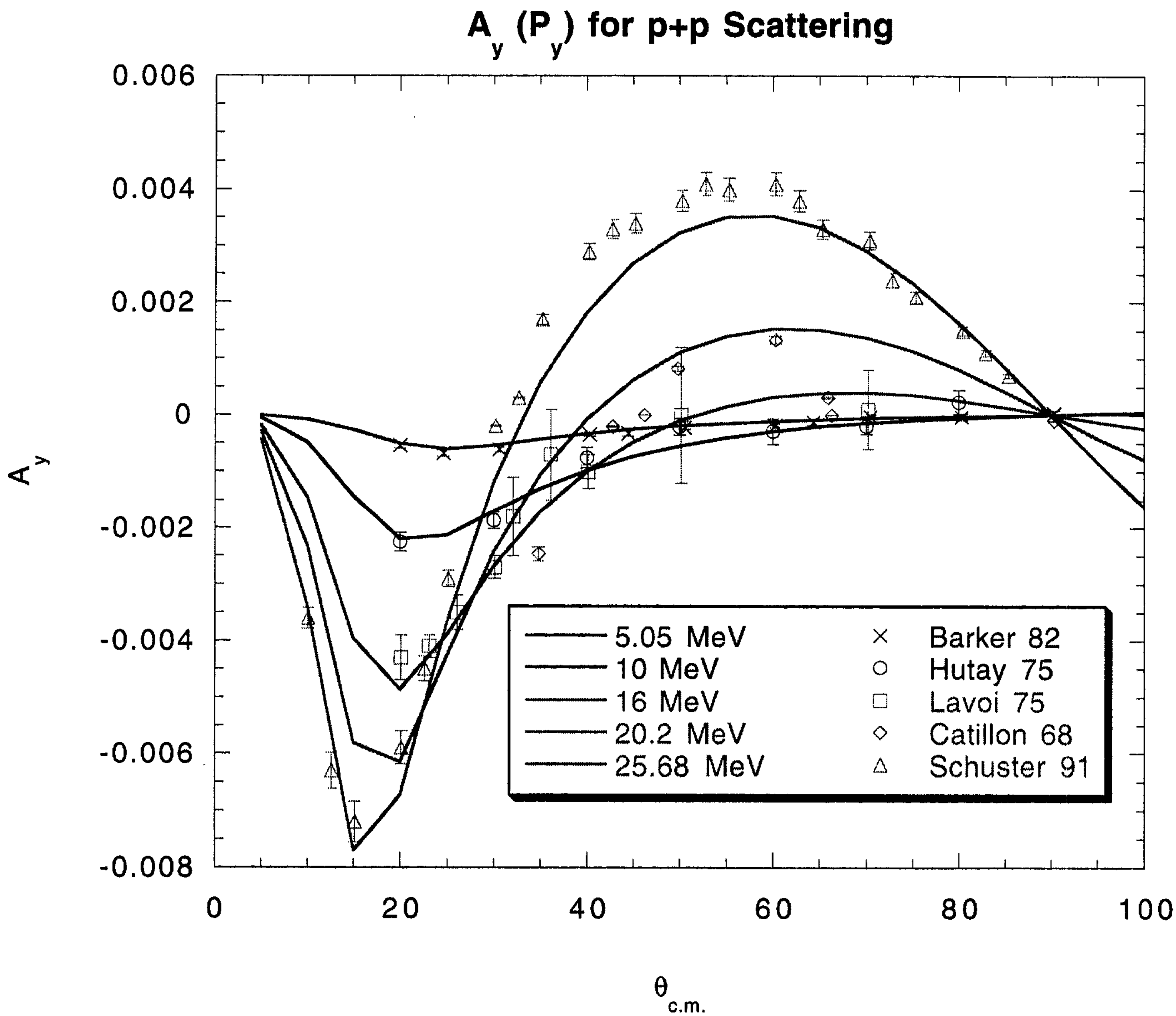
0-30 MeV N-N Analysis

Channel	a_c (fm)	l_{max}
p-p	3.26	3
n-p	3.26	3
γ-d	150	1

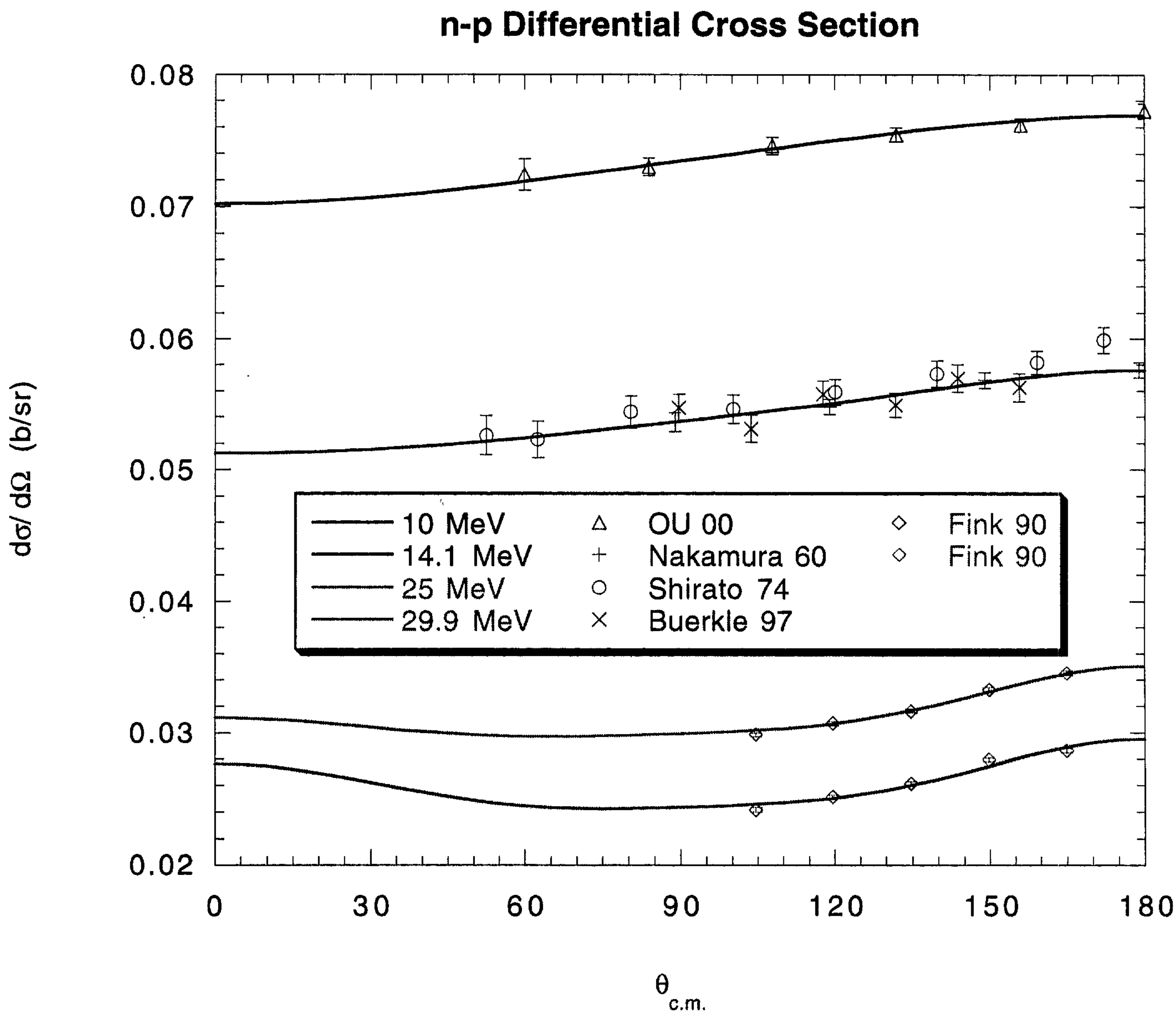
Reaction	# Pts.	χ^2	Observable Types
p(p,p)p	694	847	$\sigma(\theta)$, $A_y(p)$, $C_{x,x}$, $C_{y,y}$, $K_x^{x'}$, $K_y^{y'}$, $K_z^{x'}$
p(n,n)p	2046	2759	σ_T, $\sigma(\theta)$, $A_y(n)$, $C_{y,y}$, $K_y^{y'}$
p(n,γ)d	36	65	σ_{int}, $\sigma(\theta)$, $A_y(n)$
d(γ,n)p	27	25	σ_{int}, $\sigma(\theta)$
Norms.	128	75	
Total	2931	3771	17

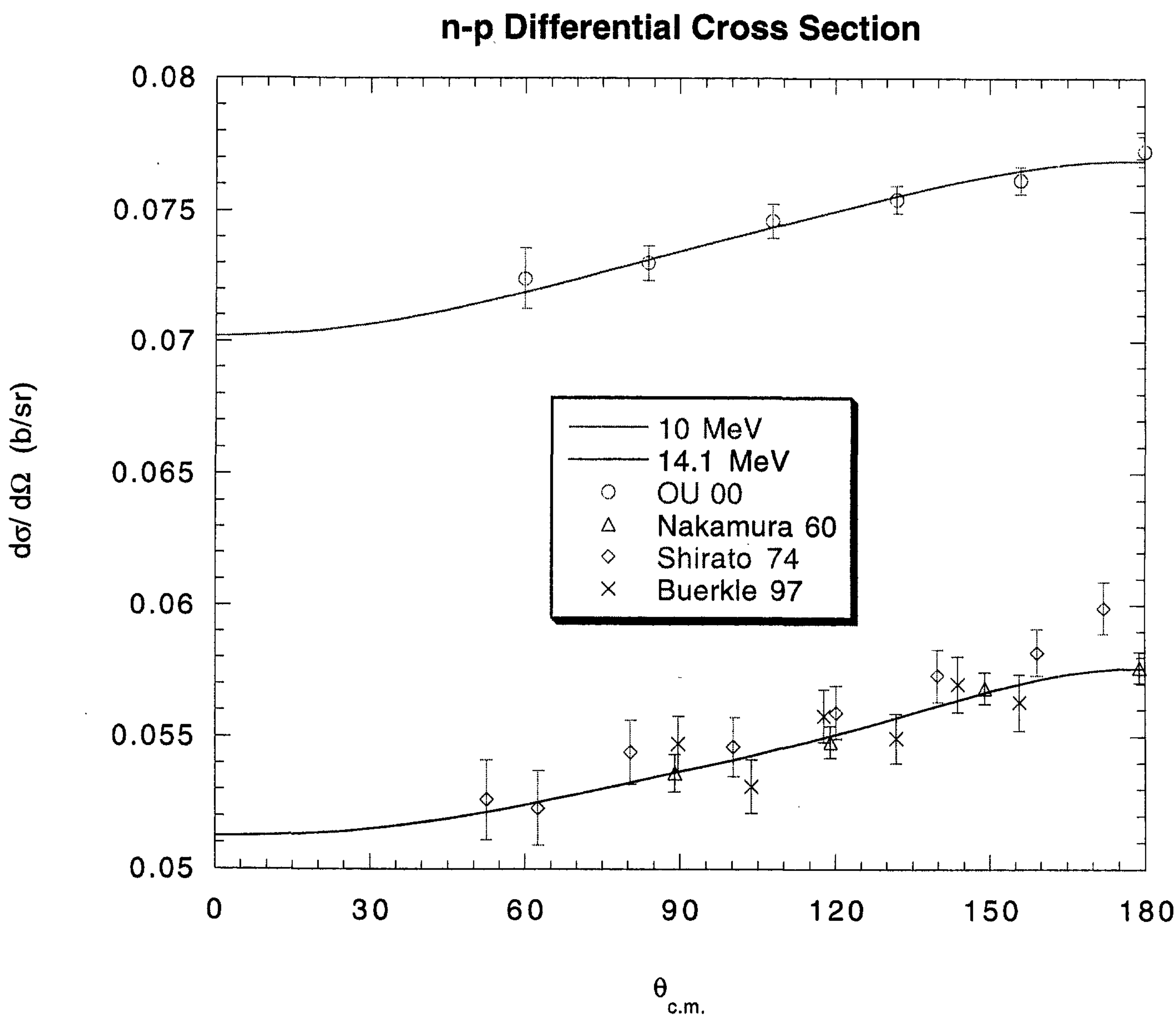
free parameters = 46+128 $\Rightarrow$ χ^2/degree of freedom = 1.37


p+p Differential Cross Section
100
10
1
0.1
0.01
0
30
60
90
120
150
180
dσ/ dΩ (b/sr)
θ cm
4.98 MeV
9.92 MeV
14.16 MeV
19.84 MeV
25.6 MeV
Imai 75
Hegland 76
Kikuchi 60
Royden 59
Jeong 60

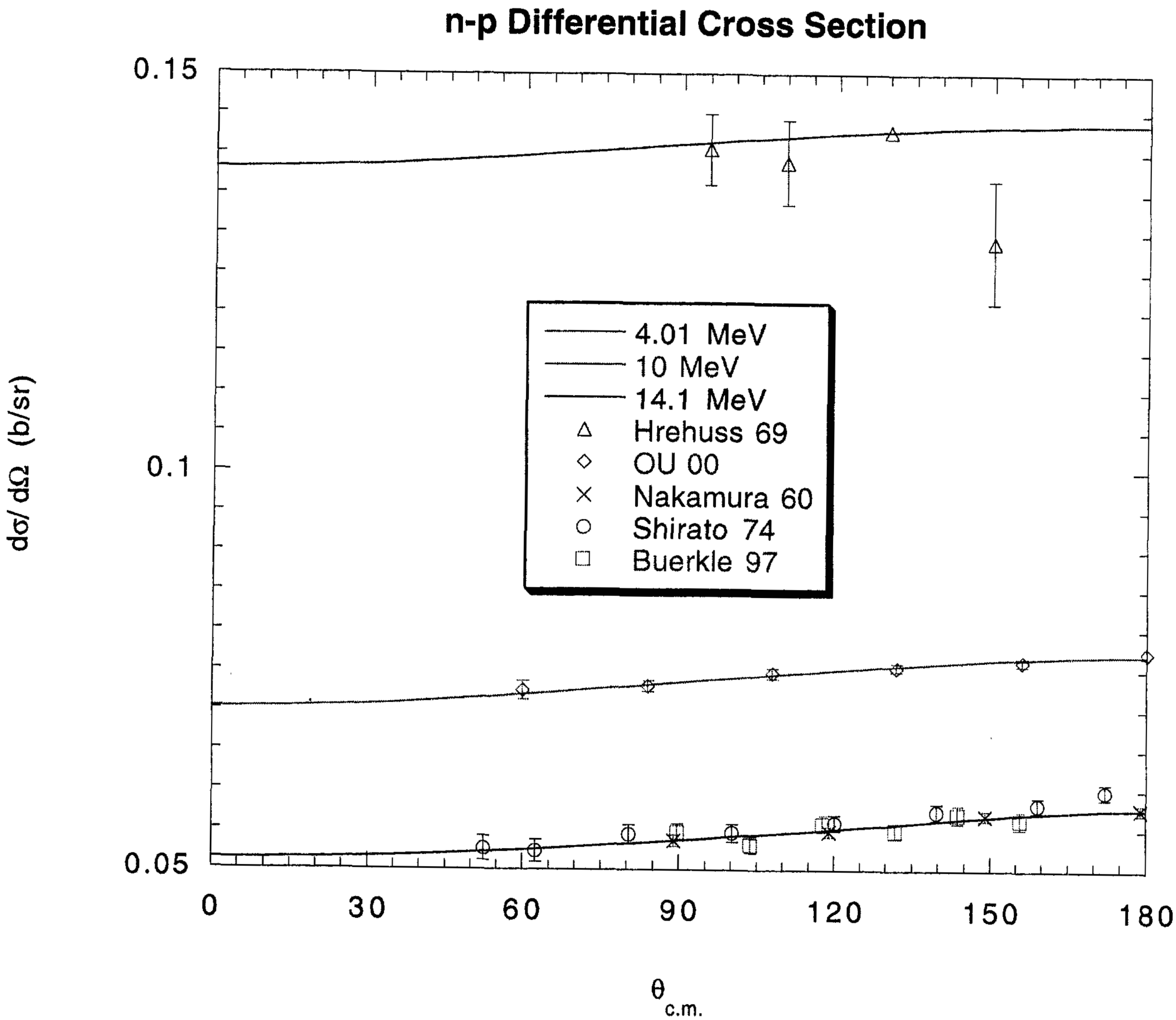


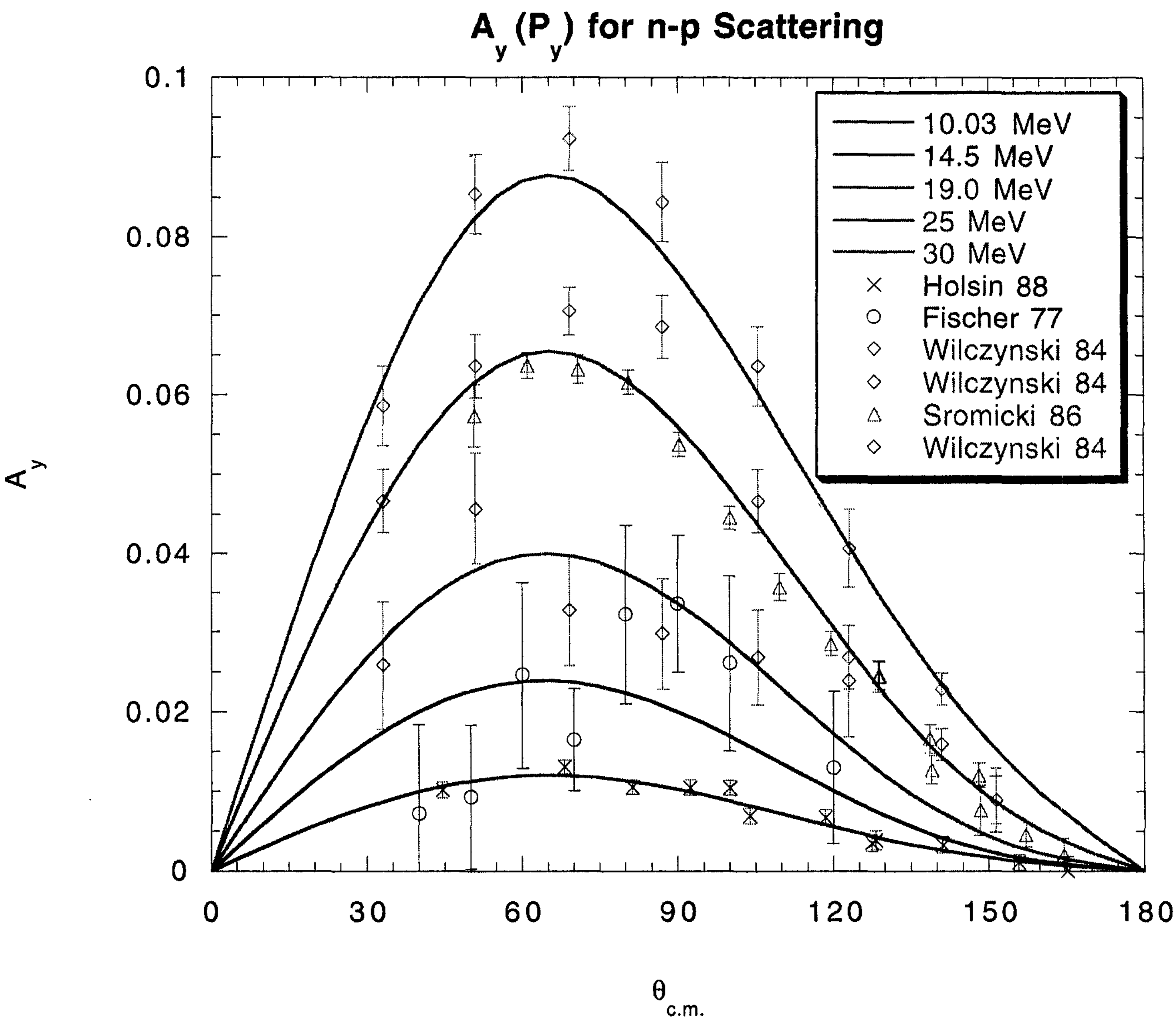
Ay (Py) for p+p Scattering
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
Ay
0
20
40
60
80
100
θc.m.
5.05 MeV
10 MeV
16 MeV
20.2 MeV
25.68 MeV
Barker 82
Hutay 75
Lavoi 75
Catillon 68
Schuster 91

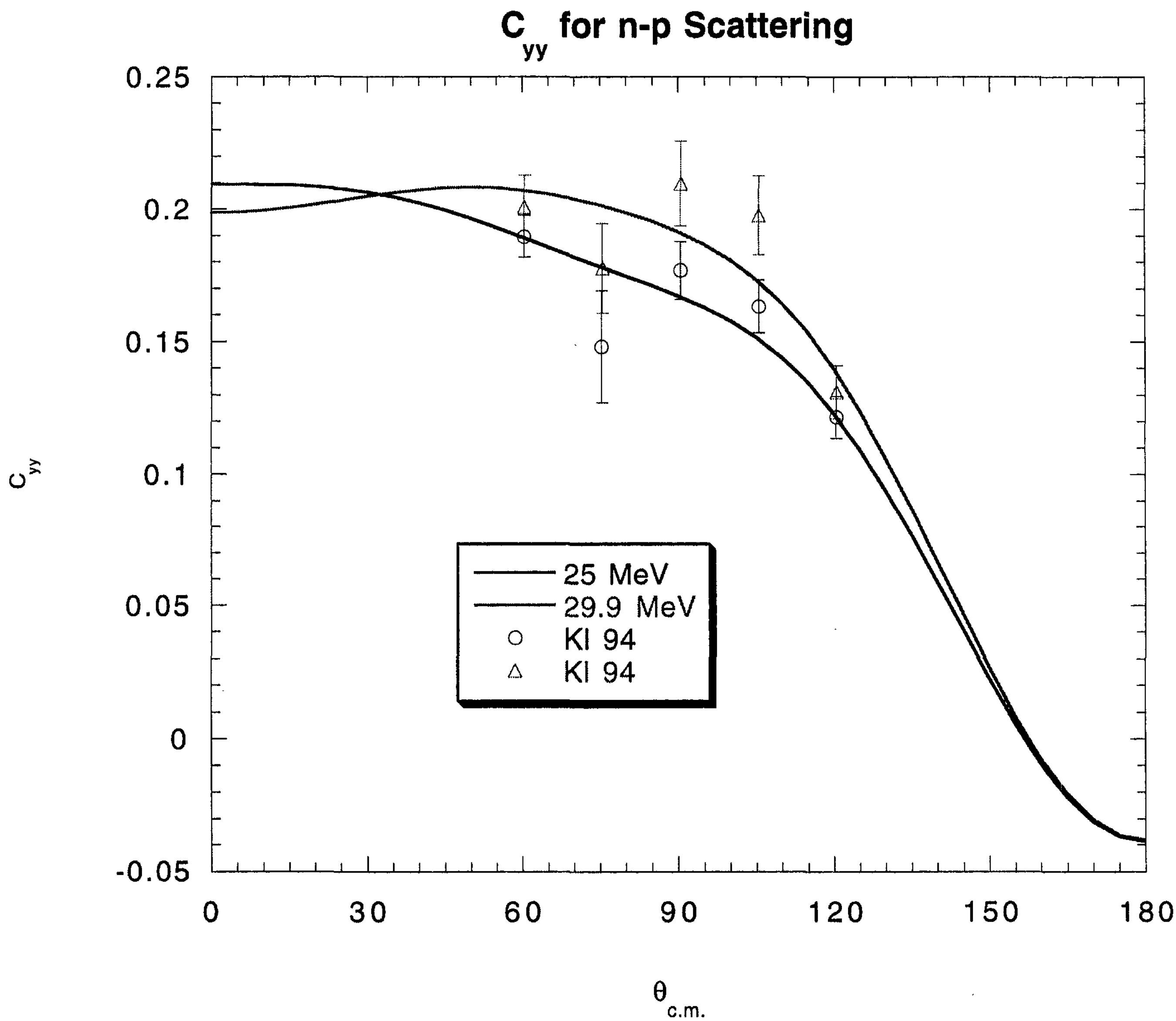

n-p Differential Cross Section
0.08
0.07
0.06
0.05
0.04
0.03
0.02
dσ/ dΩ (b/sr)
10 MeV
14.1 MeV
25 MeV
29.9 MeV
OU 00
Nakamura 60
Shirato 74
Buerkle 97
Fink 90
Fink 90
0
30
60
90
120
150
180
θ c.m.



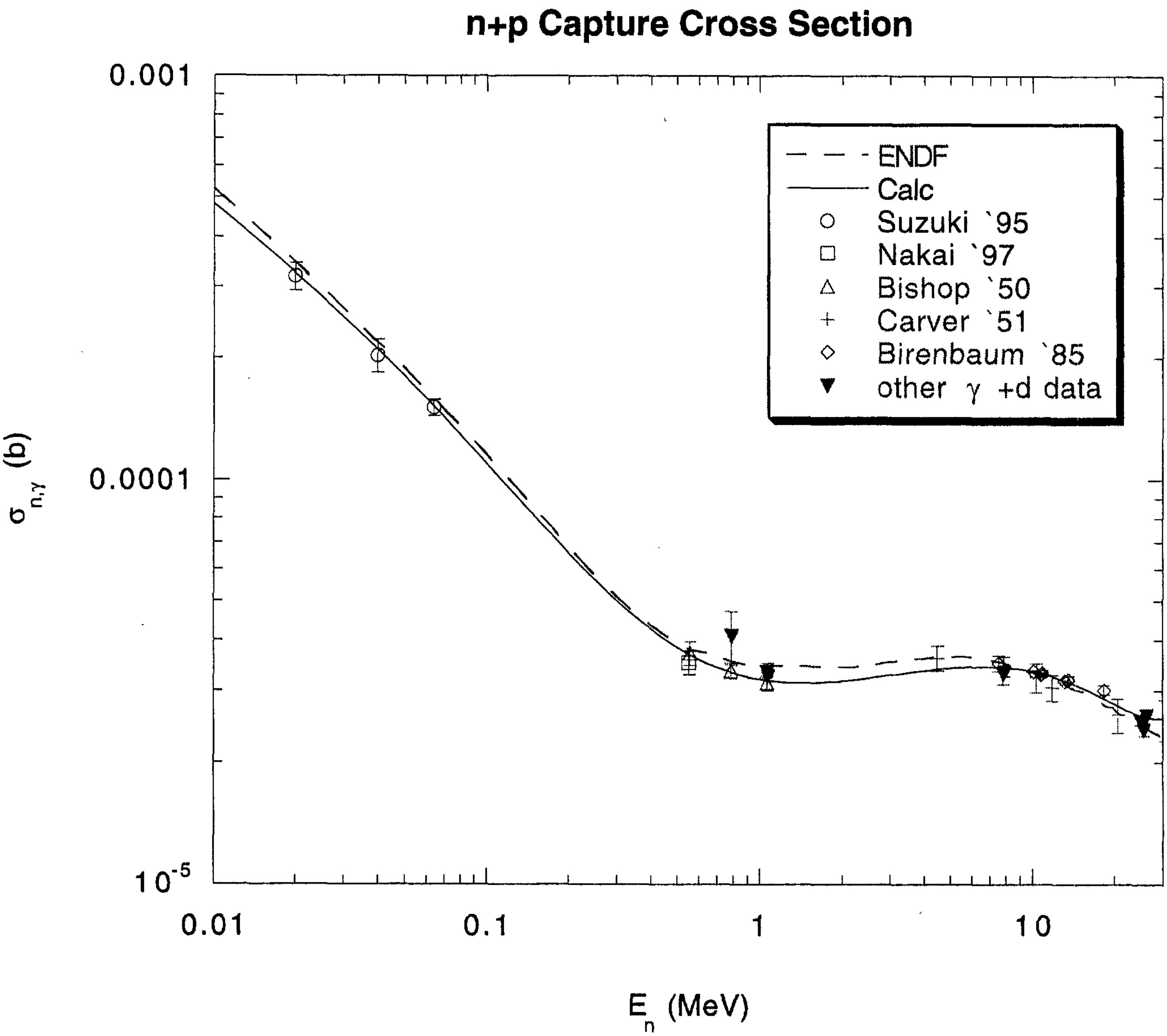
n-p Differential Cross Section
0.08
0.075
0.07
0.065
0.06
0.055
0.05
dσ/ dΩ (b/sr)
10 MeV
14.1 MeV
OU 00
Nakamura 60
Shirato 74
Buerkle 97
0
30
60
90
120
150
180
θ c.m.

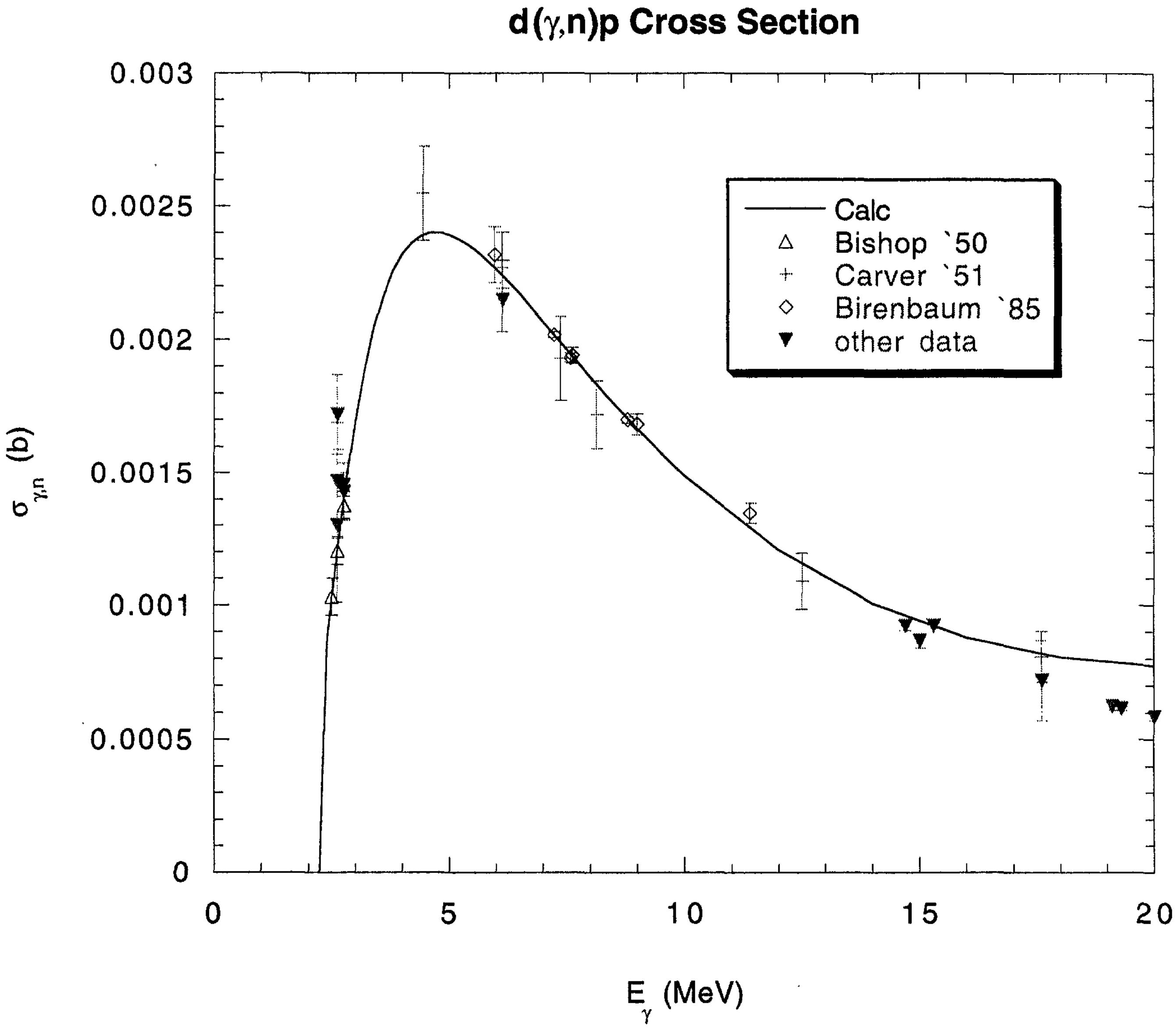

n-p Differential Cross Section
0.15
0.1
0.05
dσ/ dΩ (b/sr)
0
30
60
90
120
150
180
θ c.m.
4.01 MeV
10 MeV
14.1 MeV
Hrehuss 69
OU 00
Nakamura 60
Shirato 74
Buerkle 97


A_y (P_y) for n-p Scattering
0.1
0.08
0.06
0.04
0.02
0
A_y
0
30
60
90
120
150
180
θ_c.m.
10.03 MeV
14.5 MeV
19.0 MeV
25 MeV
30 MeV
Holsin 88
Fischer 77
Wilczynski 84
Wilczynski 84
Sromicki 86
Wilczynski 84

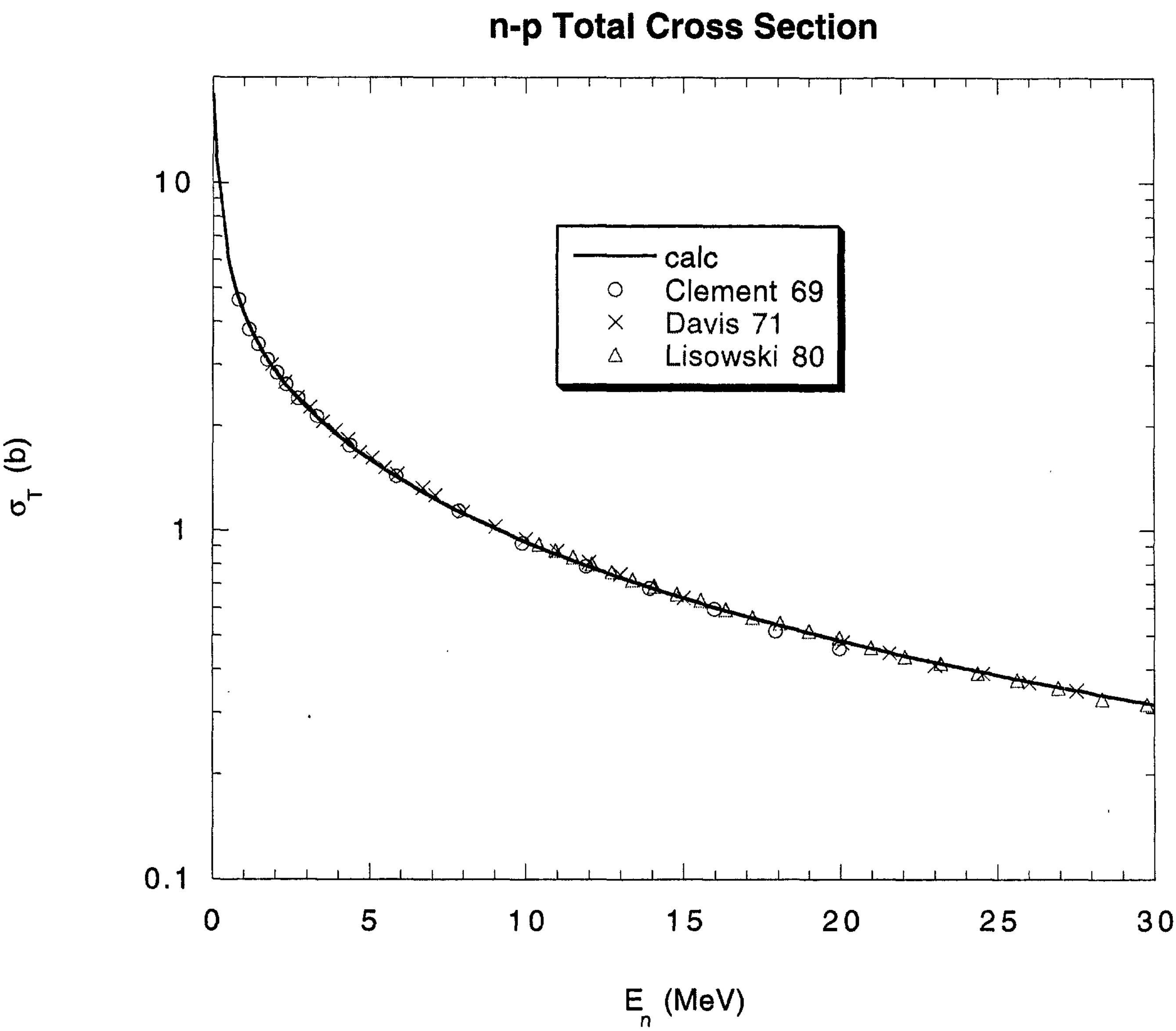


C_{yy} for n-p Scattering
0.25
0.2
0.15
0.1
0.05
0
-0.05
C_{yy}
0
30
60
90
120
150
180
$\theta_{c.m.}$
25 MeV
29.9 MeV
KI 94
KI 94

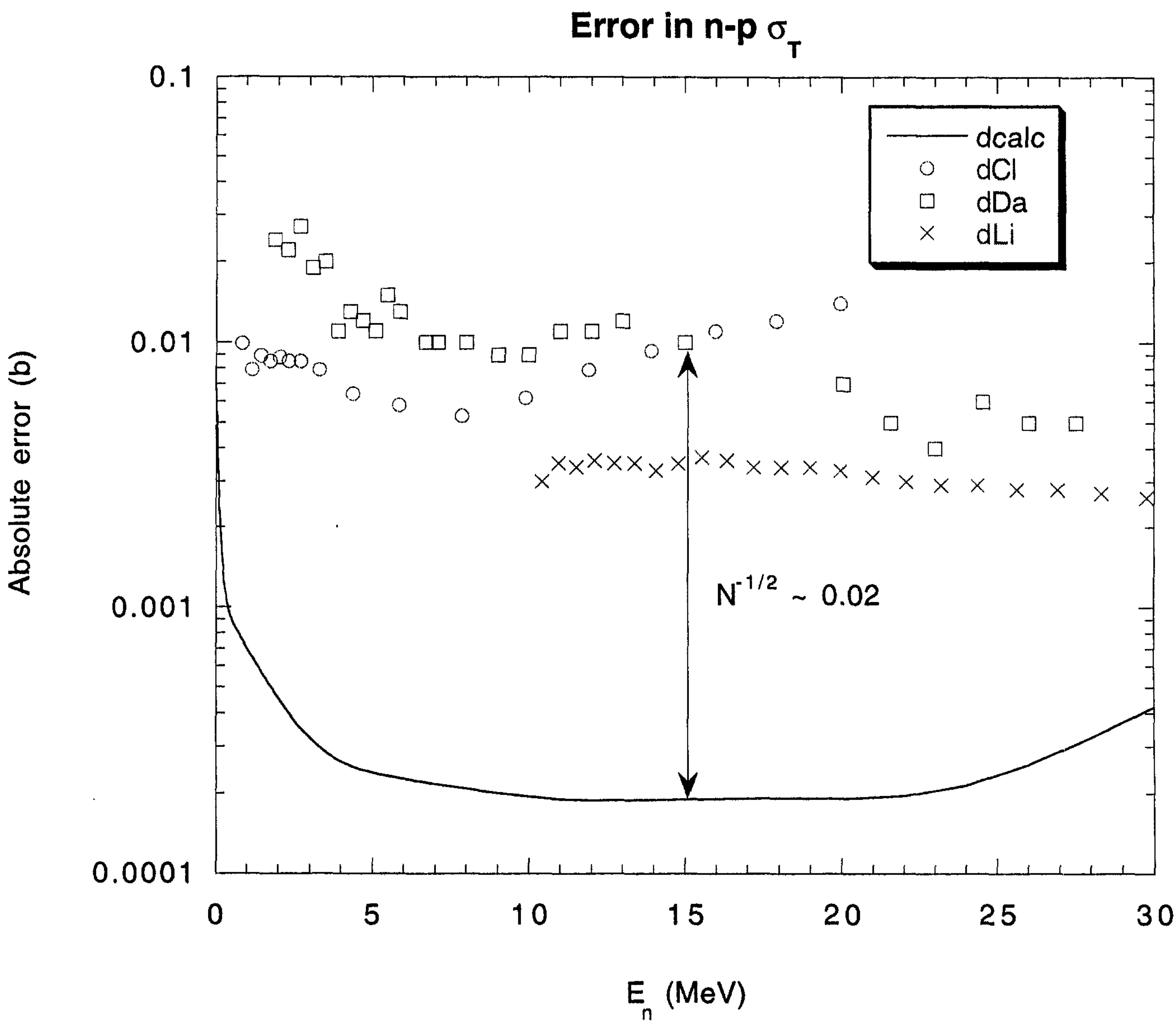

n+p Capture Cross Section
ENDF
Calc
Suzuki `95
Nakai `97
Bishop `50
Carver `51
Birenbaum `85
other γ +d data
0.001
0.0001
10^-5
0.01
0.1
1
10
σ_n,γ (b)
E_n (MeV)



d(γ,n)p Cross Section
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
0
5
10
15
20
$\sigma_{\gamma,n}$ (b)
E_γ (MeV)
Calc
Bishop `50
Carver `51
Birenbaum `85
other data

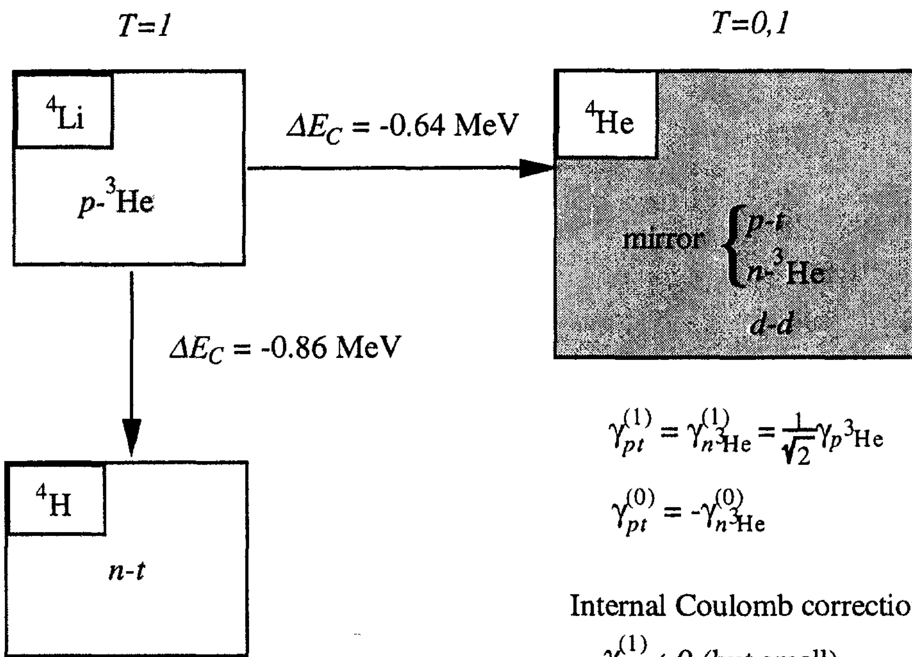


n-p Total Cross Section
calc
Clement 69
Davis 71
Lisowski 80
10
1
0.1
σT (b)
0
5
10
15
20
25
30
En (MeV)



Error in n-p σ_T
0.1
0.01
0.001
0.0001
Absolute error (b)
dcalc
dCl
dDa
dLi
$N^{-1/2} \sim 0.02$
0
5
10
15
20
25
30
E_n (MeV)

Charge-independent A = 4 R-matrix Analysis



$\gamma_{pt}^{(1)} = \gamma_{n\,^3\mathrm{He}}^{(1)} = \frac{1}{\sqrt{2}}\gamma_{p\,^3\mathrm{He}}$

$\gamma_{pt}^{(0)} = -\gamma_{n\,^3\mathrm{He}}^{(0)}$

Internal Coulomb correction:

$\gamma_{dd}^{(1)} \neq 0$ (but small)

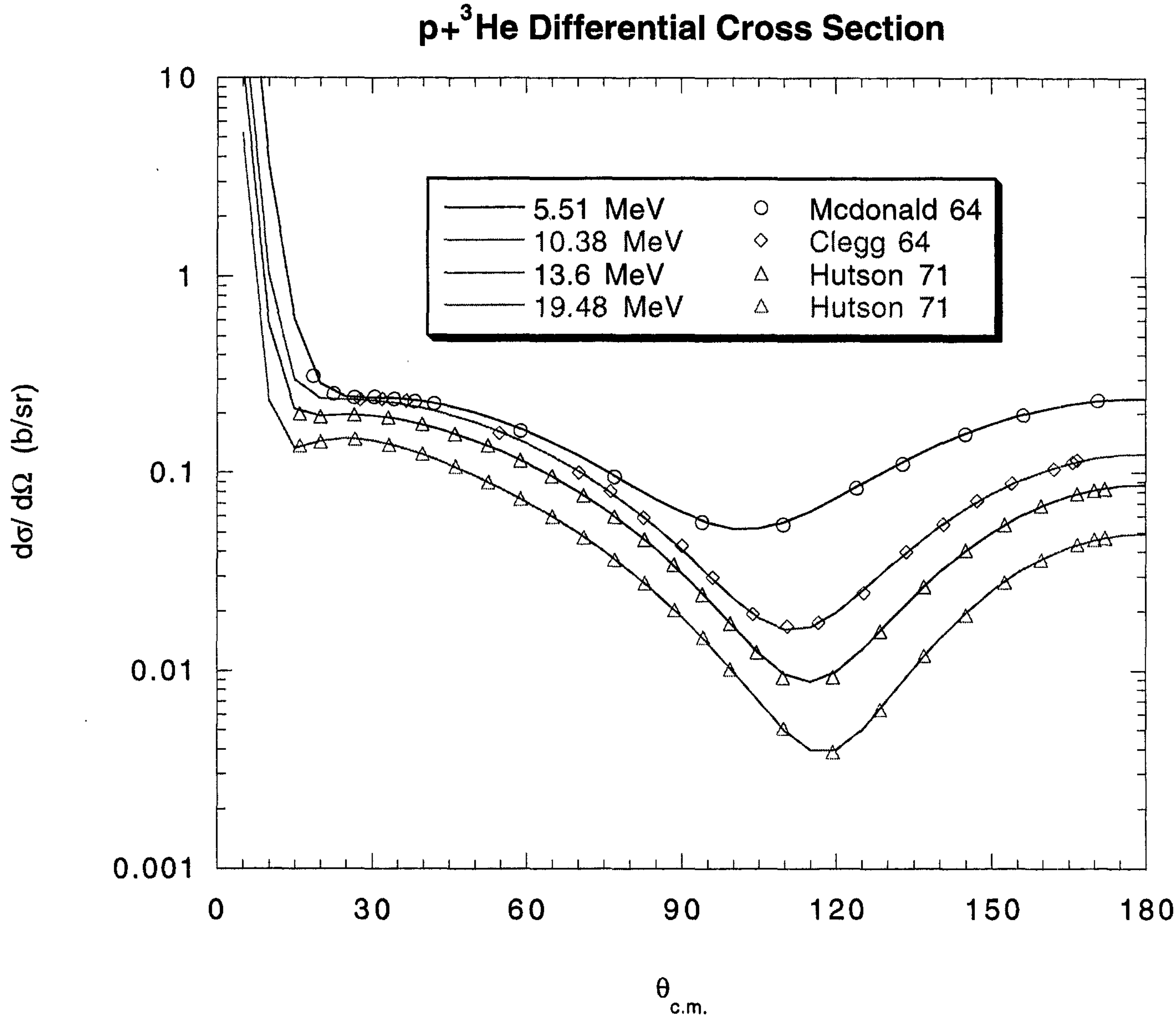
$\gamma_{nt} = \gamma_{p\,^3\mathrm{He}}$

0- 20 MeV A=4, T=1 Analysis

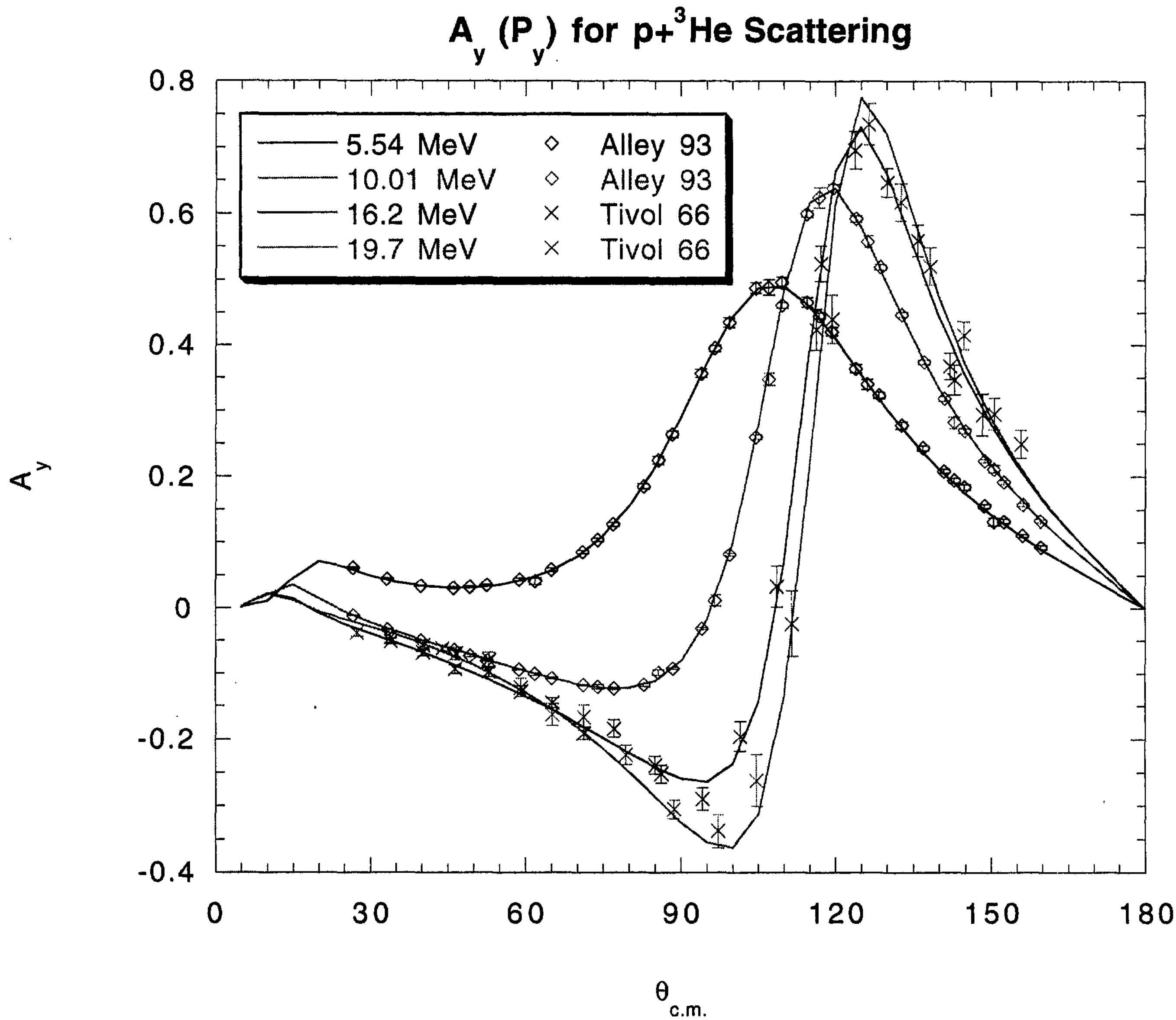
Channel	a_c (fm)	l_{max}
p-^{3}He	4.9	3
n-^{3}H	4.9	3

Reaction	# Pts.	χ^2	Observable Types
^{3}He(p,p)^{3}He	1303	1616	$\sigma(\theta)$, $A_y(p)$, $A_y(^3He)$, $C_{x,x}$, $C_{y,y}$, $C_{x,z}$, $C_{z,x}$, $C_{z,z}$, $K_x^{x'}$, $K_y^{y'}$, $K_z^{x'}$
^{3}H(n,n)^{3}H	144	591	σ_T
Norms.	4	5	
Total	1451	2212	12

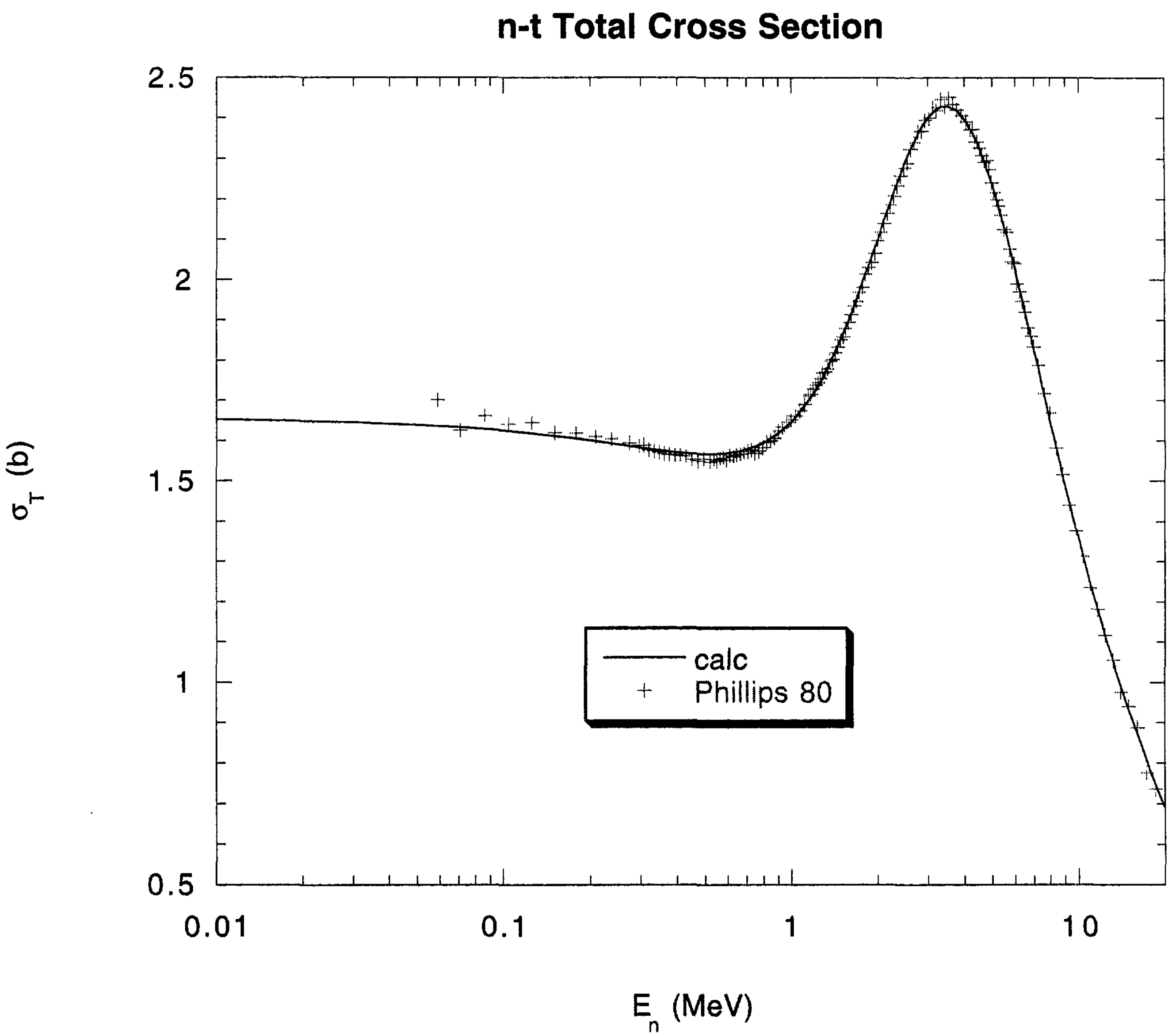
free parameters = 58+4 $\Rightarrow$ χ^2/degree of freedom = 1.59

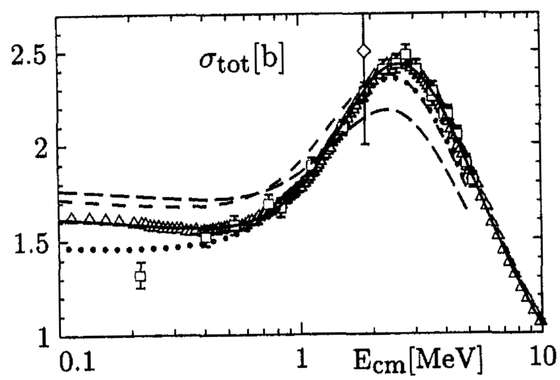


p+^{3}He Differential Cross Section
10
1
0.1
0.01
0.001
0
30
60
90
120
150
180
dσ/ dΩ (b/sr)
θ c.m.
5.51 MeV
10.38 MeV
13.6 MeV
19.48 MeV
Mcdonald 64
Clegg 64
Hutson 71
Hutson 71

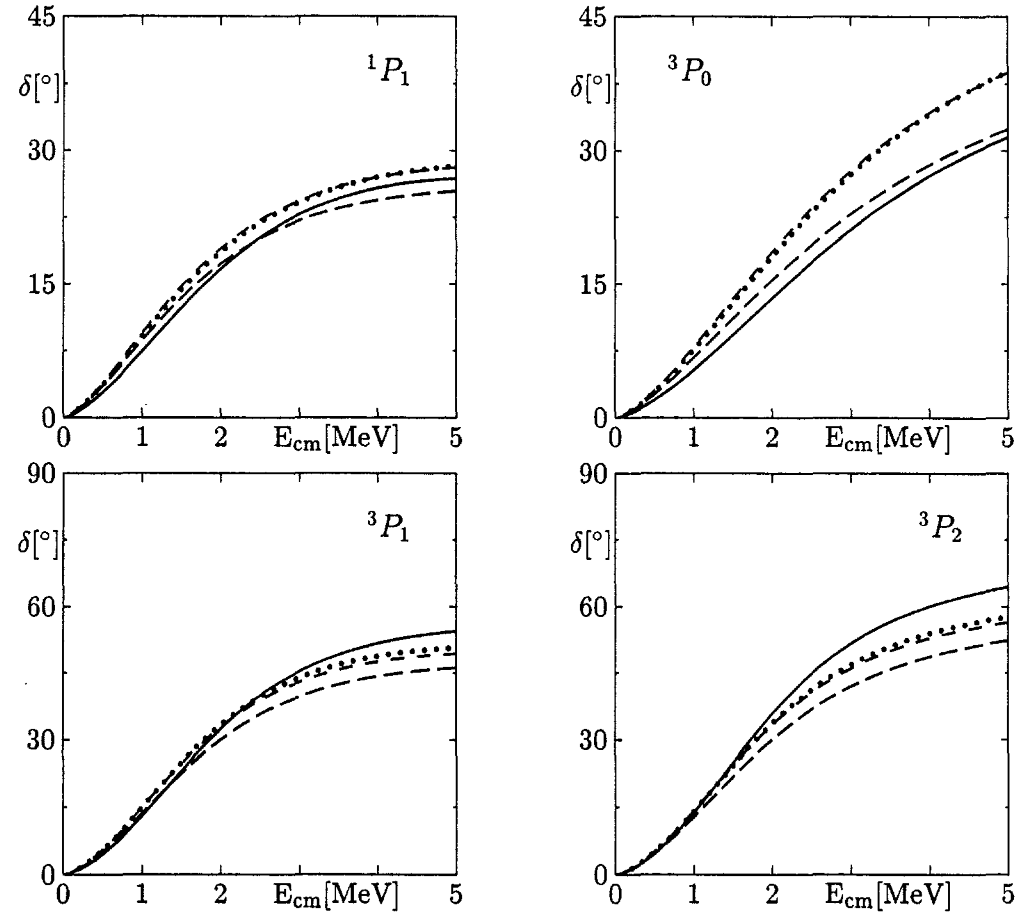


A_y (P_y) for $p+{}^3He$ Scattering
5.54 MeV
10.01 MeV
16.2 MeV
19.7 MeV
Alley 93
Alley 93
Tivol 66
Tivol 66
A_y
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
30
60
90
120
150
180
$\theta_{c.m.}$


n-t Total Cross Section
2.5
2
1.5
1
0.5
0.01
0.1
1
10
σ_T (b)
E_n (MeV)
calc
Phillips 80



σ_{tot}[b]
2.5
2
1.5
1
0.1
1
E_{cm}[MeV]
10



R-matrix (——), Av_{18} (- - -), Av_{18}+UIX (····), Av_{18} J_3^+ only (— —)

^{4}He System Analysis

Channel	l_{max}	a_c (fm)
p-t	3	4.93
n-^{3}He	3	4.93
d-d	3	7.00

Reaction	Energy Range	# Observable Types	# Data Points
T(p,p)T	E_p=0-11 MeV	3	1382
T(p,n)^{3}He + inv.	E_p=0-11 MeV	5	726
^{3}He(n,n)^{3}He	E_n=0-10 MeV	2	126
D(d,p)T	E_n=0-10 MeV	6	1382
D(d,n)^{3}He	E_d=0-10 MeV	6	700
D(d,d)D	E_d=0-10 MeV	6	336
	Totals:	28	4652

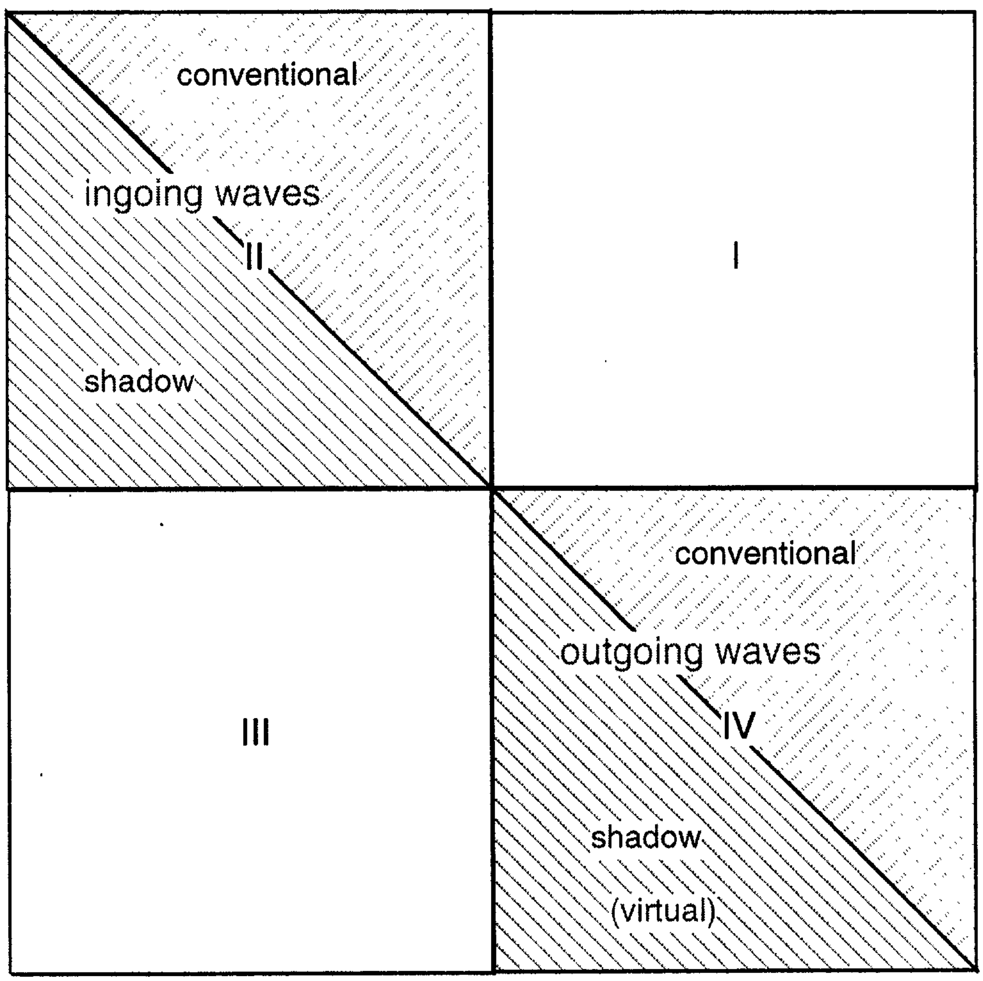
Properties of Quasistationary Decaying States in the Complex Momentum Plane

$$\psi_\mu^{q.s.}(r_c,t) = (r_c|\mu\rangle\, \mathrm{e}^{-iE_\mu t}$$

$$E_\mu = E_{r\mu} - \tfrac{1}{2} i\Gamma_\mu, \quad (\Gamma_\mu > 0)$$

$$(r_c|\mu\rangle \to \left(\frac{\Gamma_{c\mu}}{2k_{c\mu}a_c}\right)^{\frac{1}{2}} \mathrm{e}^{ik_{c\mu}r_c} = g_{c\mu}\,\mathrm{e}^{ik_{c\mu}(r_c-a_c)}$$

$$\Gamma_\mu(\mu|\mu) = 2\sum_c |g_{c\mu}|^2 \Im[L_c(k_{c\mu})]$$



(Erlangen talk)

Summary and Outlook

- Multichannel R-matrix parametrizations contain much useful information about light nuclei. At physical energies, one obtains smoothed interpolations and extrapolations of the experimental data, as well as energy-dependent S-matrix elements for all (two-body) reactions.
- A relatively straightforward continuation of the S matrix to complex energies gives information about the resonances in light systems.
- Three-body-force effects appear to be even more striking in the A=4 system than in A=3. Conventional three-body interactions cannot account for the differences seen between the calculations including two-body forces only and P-wave phase shifts extracted from the experimental data.
- Further work on the A=3 system will provide high-precision S-matrix elements to compare with microscopic calculations. Nucleon exchange effects are very important for N+d scattering, and may contribute to the virtual state seen in the J=1/2 S-wave.